In search of the right middleman

Utpal Bhattacharya [*†]

University of Iowa, 509 PHBA, Iowa City, IA 52242, USA

Abdullah Yavas

Penn State University, 409 BAB, University Park, PA 16802, USA

Abstract

This paper shows that when traders go to a middleman only if they cannot get a better price somewhere else, the two popular evaluation criteria for middlemen – low bid-ask spreads and low inventory holding costs – may be inconsistent.

1. Introduction

In contemporary trading environments, where prospective buyers and sellers increasingly use new technology to bypass the middleman and trade directly with each other if they can get a better price, the middleman has tended to become a 'trader of last resort'. [1] This trend has profound implications for the institution of the Exchange. The purpose of this paper is to analyze one such important implication: How do we evaluate middlemen in such a trading environment? This question is important for any exchange that has a designated middleman, [2] and it is a question that has not received much attention in the literature.

In the New York Stock Exchange, for example, the overall criteria that the exchange uses to evaluate an agent before assigning him the role of a middleman is his 'ability to maintain an orderly and efficient trading system and to stabilize prices'. Two popular measures for this are how low a bid–ask spread the middleman can set for a given order flow, and how well can he absorb occasional order imbalances so as to keep prices stable. Formal models of market microstructure [see Stoll (1985) for a succinct summary] have argued that middlemen with low inventory holding

---

[1] Consider the New York Stock Exchange. Schwartz and Shapiro (1990) report that large block trading – trading that usually occurs in large blocks and is crossed in the 'upstairs market' away from the specialist (the middleman) – accounted fro 51.5% of total trading in 1989. This was a dramatic increase from the paltry 3.1% estimated in 1965.

[2] A recent story in the financial press ('U.S. Healthcare, Big Board Split Over Specialist', *Wall Street Journal*, 19 September, 1991) documented that a particular firm was so upset with the middleman assigned by the New York Stock Exchange to make a market in its security that it decided to go back to the NASDAQ exchange.

costs – such middlemen have a low coefficient of absolute risk aversion, either because they are less risk averse or they have more wealth – also set low bid–ask spreads.

The major objective of this paper is to show that these two popular criteria *may* be inconsistent. The intuition for this result is the following: the higher the bid–ask spread set by the middleman, the harder traders will search to avoid having to go to the middleman. The resulting increase in search intensities *may* result in more order imbalances. If this is the case, then a middleman with a good ability to absorb these imbalances will be willing to risk the chance of them occurring by increasing his spread.

The paper is organized as follows. Section 2 presents a parameterized bilateral search economy, section 3 characterizes the equilibrium, and section 4 concludes.

2. A parametric bilateral search economy

Intermediaries are useful because, by their existence, they decrease the search costs we would normally have to incur to fund each other to trade. Aspects of this insight have been explored by Demsetz (1968), Townsend (1978), Rubinstein and Wolinsky (1987), Gehrig (1990) and Yavas (1991a, 1991b, 1992). Our model focuses more on the institutional features.

At date $t = 0$, the exchange decides on the 'right' middleman from amongst M potential risk-neutral middlemen to make a market in a particular security. These middlemen are identical in all respects except on one critical dimension; they differ in their ability to absorb order imbalances. Specifically, we assume that the kth middleman, where $k \in \{1, 2, \ldots, M\}$, has an order imbalance cost of $c_k |x|$. Here $|x|$ is the absolute change in his inventory level from his desired inventory position (which we normalize to zero), and c_k is a measure of his ability to absorb that change. $c_k \in [0, C]$, and the lower the c_k, the better is the kth middleman's ability to absorb order imbalances. It is assumed that these costs are common knowledge.

At date $t = 1$ the designated middleman acts like a leader in a Stackelberg game and sets his bid (β) and ask (α) prices to maximize his surplus, subject to a zero expected inventory change constraint. While setting his bid and ask prices, he takes into account the fact that, expost, a costly inventory imbalance might occur and that the amount of this imbalance will depend on the bid–ask spread that he sets now; this is where his capability to handle inventory imbalances will prove useful. He also takes into account the reaction functions of the traders that he might potentially trade with.

At date $t = 2$, four traders realize their private valuations of a security – V_1, V_2, V_3 and V_4. These lie uniformly spaced on the real line. Without loss of generality, we normalize these to equal -1.5, -0.5, $+0.5$ and $+1.5$, respectively. This ensures that $\alpha - 0 = 0 - \beta$. The traders then decide on whether to seek out each other for a mutually beneficial trade. If they meet each other, we assume that they split the surplus equally. Search, however, is costly and inefficient. The probability that trader i finds trader j, $\theta_{i,j}/6$, is $(S_i + S_j)/6$, where S_i and S_j are their respective search intensities. $S_i, S_j \in [0, 1]$. This specification ensures that if everyone searches their hardest – search intensities are unity – they will meet someone. The cost of search for each trader i is $Q(S_i) = \gamma S_i^2$. The parameter γ is a measure of the efficiency of search for each trader and, hence, a metric for the competition the middleman faces from the 'outside' search market; the larger the γ, the lower is the competition. If γ is infinity, the middleman does not face any competition. We impose a restriction on the exogenous parameter, $\gamma : \gamma \geq 1/3$. This ensures that, in equilibrium, the sum of all the endogenous probabilities, $\theta_{i,j}/6$, is less than unity.

Finally, at $t = 3$ the traders who had decided to search for each other at $t = 2$ and who could not find anyone or who met someone who has not willing to give them a better price than the

middleman, are allowed to go back to the middleman without being penalized. This formalizes the notion of the middleman as the 'trader of last resort', whom one could always go to if one did not find a better bargain somewhere else. By the rules of the exchange, the middleman has to kept his market open always for anyone who wants to trade. [3]

3. Characterization of equilibrium

The Stackelberg game has to be solved backwards. As traders can always go back to the middleman at $t = 3$ without incurring a penalty, and since there is always the possibility of finding a favorable match and getting a good bargain at $t = 2$ if they search or wait to be contacted, going to the middleman only at $t = 3$ is a weakly dominant strategy. Hence, it is only at $t = 3$ that the middleman is approached. So at $t = 2$ the traders maximize their surplus and choose their search intensities, based on their individual valuations and the bid and ask prices that they observe. As the traders can still trade with the middleman after searching and finding someone at $t = 2$, the surplus that they can get by trading with the middleman will constitute a threat point in their bargaining process with the other party. Hence, if two traders meet, they will transact if and only if they get a bigger surplus than they would otherwise get by trading with the middleman. If they do transact, then by this bargaining rule each party first gets the surplus she would get by trading with the middleman, and the remaining surplus is shared equally. At $t = 1$, the middleman sets his bid and ask prices based on his conjectured reaction functions of the traders.

We now state the equilibrium of this parametrized example.

Proposition. If we define the function $f(c_k, \gamma)$ as

$$f(c_k, \gamma) = \frac{60\gamma - 6.25c_k}{108\gamma - 30c_k},$$ (1)

then the kth middleman, if designated as the middleman, would make the following expected surplus:

$$[2 - 1/(9\gamma)],$$ *for $1/3 \leq \gamma \leq f(c_k, \gamma)$ and*

$$[\{3 - 2/(3\gamma)\} - \{(2.5c_k)(1 - 2.5/(12\gamma))/(9\gamma)\}],$$ *for $\gamma > f(c_k, \gamma)$,*

and he would set the following bid–ask spreads:

$$V_2 + \epsilon, V_3 - \epsilon,$$ *for $1/3 \leq \gamma \leq f(c_k, \gamma)$ and*

$$V_1 + \epsilon, V_4 - \epsilon,$$ *for $\gamma > f(c_k, \gamma)$,*

and the optimal search intensity of trader i is

$$S_1 = S_2 = S_3 = S_4 = 1/(12\gamma),$$ *for $1/3 \leq \gamma \leq f(c_k, \gamma)$ and*

$$S_1 = S_4 = 1/(4\gamma); \quad S_2 = S_3 = 1/(6\gamma),$$ *for $\gamma > f(c_k, \gamma)$*

Proof. See the appendix.

---

[3] Strictly, this is not true. In a study analyzing all trading suspensions in the NYSE during 1974–1988, Bhattacharya and Spiegel (1991) find that on average, there are four trading suspensions every day.
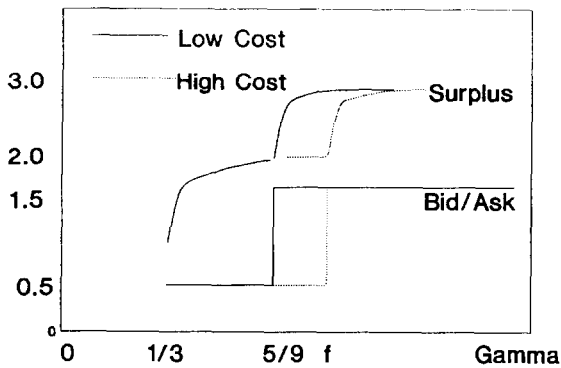
Fig. 1. Expected surpluses and bid–ask spreads.

The expected surplus for two middlemen – one with zero inventory holding cost (the unbroken line) and the other with a positive inventory holding cost (the broken line) – is plotted in Fig. 1. This figure illustrates the main idea of this paper.

Figure 1 shows us that for all values of γ less than $5/9$, both middlemen are making maximum surpluses if they set their bid and ask prices at $V_2 + \epsilon$ and $V_3 - \epsilon$, and the surpluses are the same for both. So they set their bid–ask spreads there. However, when $5/9 \leq \gamma \leq f(c_k, \gamma)$, the middleman with zero inventory holding cost finds that he will make more expected surpluses if he widens his bid–ask spread to $V_1 + \epsilon$ and $V_4 - \epsilon$, whereas the middleman with the higher inventory holding costs finds it optimal to continue maintaining his bid–ask spread at $V_2 + \epsilon$ and $V_3 - \epsilon$. When $\gamma > f(c_k, \gamma)$, the latter middleman finds it optimal to widen his bid–ask spread to $V_1 + \epsilon$ and $V_4 - \epsilon$ as well.

So it is in the region $5/9 \leq \gamma \leq f(c_k, \gamma)$ that we find that a middleman with lower inventory holding costs (costs are zero) is actually setting a higher bid–ask spread ($V_1 + \epsilon$ and $V_4 - \epsilon$) than the bid–ask spread ($V_2 + \epsilon$ to $V_3 - \epsilon$) a middleman with a higher inventory holding cost is setting.

4. Conclusion

This paper presents an example of intermediated markets where the middleman is modeled as the 'trader of last resort'; potential traders approach him only when they cannot find a better bargain somewhere else. An increase in the middleman's quoted bid–ask spread, therefore, intensifies search among traders, and may result in more order imbalances. A middleman with lower inventory holding costs can absorb these imbalances at a lower cost; hence, he will be willing to risk the chance of them occurring by increasing his quoted bid–ask spread.

What then should be the criteria for choosing the 'right' middleman? That remains an open question, and a topic for further research.

Appendix

We will do the proof in two parts. First we will restrict the bid to be between V_2 and 0, then we will consider the case of the bid between V_1 and V_2, and finally we will synthesize the two. Notice, that by symmetry, this takes care of the ask price as well. Since we are now just considering the left-hand side of the origin, we will ignore the negative signs.

Part 1: Bid between V_2 and 0 (ask between 0 and V_3)

Expected surplus of trader 1 (trader 4 will be similar by symmetry):

$1/3[0.5(S_1 + S_2)(V_1 - \beta) + (1 - 0.5(S_1 + S_2))(V_1 - \beta)] + 1/3[0.5(S_1 + S_3)(V_1 - \beta + 2\beta/2)$
$+ (1 - 0.5(S_1 + S_3))(V_1 - \beta)] + 1/3[0.5(S_1 + S_4)(V_1 - \beta + 2\beta/2) + (1 - 0.5(S_1 + S_4))$
$\times (V_1 - \beta)] - \gamma S_1^2$.

So the first-order condition is $S_1 = S_4 = \beta/(6\gamma)$.

Expected surplus of trader 2 (trader 3 will be similar by symmetry):

$1/3[0.5(S_1 + S_2)(V_2 - \beta) + (1 - 0.5(S_1 + S_2))(V_2 - \beta)] + 1/3[0.5(S_2 + S_3)(V_2 - \beta + 2\beta/2)$
$+ (1 - 0.5(S_2 + S_3))(V_2 - \beta)] + 1/3[0.5(S_2 + S_4)(V_2 - \beta + 2\beta/2) + (1 - 0.5(S_2 + S_4))$
$\times (V_2 - \beta)] - \gamma S_2^2$.

So the first-order condition is $S_2 = S_3 = \beta/(6\gamma)$. Hence,

$$\theta_{i,j} = \beta/(6\gamma), \quad \forall i,j. \tag{A.1}$$

Expected profits of the middleman:

$1/3[\theta_{1,2}\theta_{3,4}4\beta + \theta_{1,2}(1 - \theta_{3,4})4\beta + (1 - \theta_{1,2})(\theta_{3,4}4\beta + (1 - \theta_{1,2})(1 - \theta_{3,4})4\beta]$ for pairing $1 - 2, 3 - 4$
$+ 1/3[\theta_{1,3}\theta_{2,4}0 + \theta_{1,3}(1 - \theta_{2,4})2\beta + (1 - \theta_{1,3})\theta_{2,4}2\beta + (1 - \theta_{1,3})(1 - \theta_{2,4})4\beta]$ for pairing $1 - 3$,
$2 - 4 + 1/3[\theta_{1,4}\theta_{2,3}0 + \theta_{1,4}(1 - \theta_{2,3})2\beta + (1 - \theta_{1,4})\theta_{2,3}2\beta + (1 - \theta_{1,4})(1 - \theta_{2,3})4\beta]$ for pairing 1
$- 4, 2 - 3$
$= (4/3 + 72/27)\beta - 12\beta^2/(27\gamma)$ by substituting from (A.1).

Note that c_k does not matter. So the first-order condition for the middleman is

$$\beta = 4.5\gamma, \text{ if } 1/12 \le \gamma \le 1/9 \quad \text{and} \quad \beta = 0.5, \text{ for } \gamma > 1/9, \tag{A.2}$$

and the corresponding surpluses are

$$9\gamma, \text{ if } 1/12 \le \gamma \le 1/9 \quad \text{and} \quad 2 - 1/(9\gamma), \text{ for } \gamma > 1/9. \tag{A.3}$$

Since we restrict $\gamma > 1/3$, we conclude from (A.2) and (A.3) that the optimals in this range are

$$\beta^* = 0.5 - \epsilon \text{ and Surplus}^* = 2 - 1/(9\gamma) \text{ and Search Intensity}^* = 1/(12\gamma). \tag{A.4}$$

Part 2: Bid between V_1 and V_2 (ask between V_3 and V_4)

Expected surplus of trader 1 (trader 4 will be similar by symmetry):

$1/3[0.5(S_1 + S_2)(V_1 - \beta + (\beta - 0.5)/2) + (1 - 0.5(S_1 + S_2))(V_1 - \beta)]$
$+ 1/3[0.5(S_1 + S_3)(V_1 - \beta + (\beta + 0.5)/2) + (1 - 0.5(S_1 + S_3))(V_1 - \beta)]$
$+ 1/3[0.5(S_1 + S_4)(V_1 - \beta + \beta) + (1 - 0.5(S_1 + S_4))(V_1 - \beta)] - \gamma S_1^2$.

So the first-order condition is $S_1 = S_4 = \beta/(6\gamma)$.

Expected surplus of trader 2 (trader 3 will be similar by symmetry):

$1/3[0.5(S_1 + S_2)(\beta - 0.5)/2 + (1 - 0.5(S_1 + S_2))0] + 1/3[0.5(S_2 + S_3)0.5$
$+ (1 - 0.5(S_2 + S_3))0] + 1/3[0.5(S_2 + S_4)(\beta + 0.5)/2 + (1 - 0.5(S_2 + S_4))0] - \gamma S_2^2$.

So the first-order condition is $S_2 = S_3 = (\beta + 0.5)/(12\gamma)$. Hence,

$$\theta_{1,4} = \beta/(6\gamma), \theta_{2,3} = (\beta + 0.5)/(12\gamma), \theta_{1,2} = \theta_{1,3} = \theta_{2,4} = \theta_{3,4} = (1.5\beta + 0.25)/(12\gamma). \tag{A.5}$$

Expected profits of the middleman:

$1/3[\theta_{1,2}\theta_{3,4}0 - \theta_{1,2}(1 - \theta_{3,4})(c_k - \beta) - (1 - \theta_{1,2})\theta_{3,4}(c_k - \beta)$
$+ (1 - \theta_{1,2})(1 - \theta_{3,4})2\beta]$ for pairing $1 - 2, 3 - 4$
$+ 1/3[\theta_{1,3}\theta_{2,4}0 - \theta_{1,3}(1 - \theta_{2,4})(c_k - \beta) - (1 - \theta_{1,3})\theta_{2,4}(c_k - \beta)$

$+ (1 - \theta_{1,3})(1 - \theta_{2,4})2\beta]$ for pairing $1 - 3, 2 - 4$
$+ 1/3[\theta_{1,4}\theta_{2,3}0 + \theta_{1,4}(1 - \theta_{2,3})0 + (1 - \theta_{1,4})\theta_{2,3}2\beta$
$+ (1 - \theta_{1,4})(1 - \theta_{2,3})2\beta]$ for pairing $1 - 4, 2 - 3$
$= \beta^2(120 - 9c_k/\gamma)/(432\gamma) - c_k(1 - 1/(48\gamma))/(\gamma 36)$ by substituting from (A.5).

So the first order condition for the middleman is

$\beta = 0.5$,

$\quad$ if $1/4 \leq \gamma \leq (120 - 9c_k/\gamma)/(864 - 72c_k/\gamma) + (6 + 1.5c_k/\gamma)/(432 - 36c_k/\gamma)$,

$\beta = (432\gamma - 36c_k)/(120 - 9c_k/\gamma) - (6 - 1.5c_k/\gamma)/(120 - 9c_k/\gamma)$,

$\quad$ if $(120 - 9c_k/\gamma)/(864 - 72c_k/\gamma) + (6 + 1.5c_k/\gamma)/(432 - 36c_k/\gamma) \leq \gamma$

$\quad \leq 3(120 - 9c_k/\gamma)/(864 - 72c_k/\gamma) + (6 + 1.5c_k/\gamma)/(432 - 36c_k/\gamma)$,

$\beta = 1.5$,

$\quad$ if $\gamma \leq 3(120 - 9c_k/\gamma)/(864 - 72c_k/\gamma) + (6 + 1.5c_k/\gamma)/(432 - 36c_k/\gamma)$. $\quad$ (A.6)

The corresponding surpluses are

$(1 - 1/(72\gamma)) - c_k(1 - 1/(12\gamma))/(9\gamma)$,

$\quad$ if $1/4 \leq \gamma \leq (120 - 9c_k/\gamma)/(864 - 72c_k/\gamma) + (6 + 1.5c_k/\gamma)/(432 - 36c_k/\gamma)$,

$(432\gamma - 36c_k - 6 + 1.5c_k/\gamma)^2/(120\gamma - 9c_k)/432 - c_k(1 - 1/(48\gamma))/(36\gamma)$

$\quad$ if $(120 - 9c_k/\gamma)/(864 - 72c_k/\gamma) + (6 + 1.5c_k/\gamma)/(432 - 36c_k/\gamma) \leq \gamma$

$\quad \leq 3(120 - 9c_k/\gamma)/(864 - 72c_k/\gamma) + (6 + 1.5c_k/\gamma)/(432 - 36c_k/\gamma)$,

$(3 - 2/(3\gamma)) - 2.5c_k(1 - 2.5/(12\gamma))/(9\gamma)$,

$\quad$ if $\gamma \leq 3(120 - 9c_k/\gamma)/(864 - 72c_k/\gamma) + (6 + 1.5c_k/\gamma)/(432 - 36c_k/\gamma)$, $\quad$ (A.7)

and the optimal search intensities are

$S_1 = S_4 = S_2 = S_3 = 1/(12\gamma)$,

$\quad$ if $1/4 \leq \gamma \leq (120 - 9c_k/\gamma)/(864 - 72c_k/\gamma) + (6 + 1.5c_k/\gamma)/(432 - 36c_k/\gamma)$,

$S_1 = S_4 = 0.6 - 1/(120\gamma)$ and $S_2 = S_3 = 0.3 - 1/(240\gamma)$,

$\quad$ if $(120 - 9c_k/\gamma)/(864 - 72c_k/\gamma) + (6 + 1.5c_k/\gamma)/(432 - 36c_k/\gamma) \leq \gamma$

$\quad \leq 3(120 - 9c_k/\gamma)/(864 - 72c_k/\gamma) + (6 + 1.5c_k/\gamma)/(432 - 36c_k/\gamma)$,

$S_1 = S_4 = 1/(4\gamma)$ and $S_2 = S_3 = 1/(6\gamma)$,

$\quad$ if $\gamma \leq 3(120 - 9c_k/\gamma)/(864 - 72c_k/\gamma) + (6 + 1.5c_k/\gamma)/(432 - 36c_k/\gamma)$. $\quad$ (A.8)

Now we have to synthesize the two parts by comparing the surplus in (A.4) with the surplus in (A.7). We find that the surplus in (A.4) dominates the surpluses in the first two regions of (A.7), and it is greater than the surplus in the third region iff $\gamma \leq f(c_k, \gamma)$, where $f(c_k, \gamma)$ is

$$f(c_k, \gamma) = \frac{60\gamma - 6.25c_k}{108\gamma - 30c_k} .$$

We thus conclude that the optimal surplus of the kth middleman is

$$[2 - 1/(9\gamma)] , \qquad\qquad\qquad \text{for } 1/3 \le \gamma \le f(c_k, \gamma) \text{ and}$$

$$[\{3 - 2/(3\gamma)\} - \{(2.5c_k)(1 - 2.5/(12\gamma))/(9\gamma)\}] , \quad \text{for } \gamma > f(c_k, \gamma) ,$$

and he would set the following bid–ask spreads:

$$V_2 + \epsilon, V_3 - \epsilon , \quad \text{for } 1/3 \le \gamma \le f(c_k, \gamma) \text{ and}$$
$$V_1 + \epsilon, V_4 - \epsilon , \quad \text{for } \gamma > f(c_k, \gamma) ,$$

and the optimal search intensity of trader i is

$$S_1 = S_2 = S_3 = S_4 = 1/(12\gamma) , \qquad \text{for } 1/3 \le \gamma \le f(c_k, \gamma) \text{ and}$$
$$S_1 = S_4 = 1/(4\gamma); \qquad S_2 = S_3 = 1/(6\gamma) , \quad \text{for } \gamma > f(c_k, \gamma) . \quad \text{Q.E.D.}$$

References

Bhattacharya, U. and M. Spiegel, 1991, Anatomy of a market failure: NYSE trading suspensions (1974–1988), Working Paper, Columbia University.

Demsetz, H., 1968, The cost of transacting, Quarterly Journal of Economics 97, 33–53.

Gehrig, T., 1990, Intermediation in search markets, Discussion Paper No. 73, Financial Markets Group, LSE, London.

Rubinstein, A. and A. Wolinsky, 1987, Middlemen, Quarterly Journal of Economics, 581–593.

Schwartz, R.A. and J.E. Shaprio, 1990, Institutionalization of equity markets: Challenges and implications, Unpublished manuscript, New York University.

Stoll, H.R., 1985, The Stock Exchange specialist system: An economic analysis, monograph series in finance and economics, Salomon Brothers Center for the Study of Financial Institutions, New York University.

Townsend, R., 1978, Intermediation with costly bilateral exchange, Review of Economic Studies, 417–425.

Yavas, A., 1991a, The role of intermediation in a search model, Working Paper, Penn State University.

Yavas, A., 1991b, A simple model of bid–ask spread and search, Working Paper, Penn State University.

Yavas, A., 1992, Market makers versus match makers, Journal of Financial Intermediation 2, 33–58.